MATH 3070
Assignment # 6
Due Thursday, November 27, 2008

1. Prove that the continued fraction expansion of $\sqrt{7}$ is $[2, \overline{1, 1, 1, 4}]$.

2. Calculate the first 7 convergents of π^2. (Up to p_6/q_6).
   Note 1: An 8-digit calculator has enough accuracy to do this. Compare this with what you can get if you use higher precision.
   Note 2: Compare with the first few convergents of $\pi = [3, 7, 15, 1, 292, 1, \ldots]$:

$$3, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{103993}{33102}, \frac{208341}{66317}, \ldots$$

   The 292 in the expansion for π gives much better approximations than the 47 in π^2.

3. Prove that if d is a perfect square, then the only solutions in integers (x, y) to the Pell equation $x^2 - dy^2 = 1$ are $(1, 0)$ and $(-1, 0)$.

4. Find the first 5 solutions to the Pell equation $x^2 - 5y^2 = 1$ in positive integers (x, y).

5. Let x be irrational. Prove that the sequence $\{x, 2x, 3x, \ldots\}$ contains no integers, but contains elements arbitrarily close to integers. To be precise, prove that for any $N \in \mathbb{N}$, we can find an n such that nx is within $1/N$ of an integer.