MATH 2112 / CSCI 2112
Assignment # 7
Due Wednesday, November 8, 2006

Section 4.4: # 2, 5, 9

Section 8.1: # 12, 16, 19c, 21c, 40

Section 8.2: # 4, 29, 13, 38, 44

Also do these problems:

1. Prove that for all $n \geq 1$,

$$F_n^2 < F_{n-1}F_{n+1}, \qquad \qquad \text{if } n \text{ is odd,}$$
$$F_{n-1}F_{n+1} < F_n^2, \qquad \qquad \text{if } n \text{ is even.}$$

2. Professor Evil, like all evil-doers, keeps a number of prisoners in his secret base at all times. There the prisoners are forced to work endlessly on math problems like this one. Every week, the professor gives the prisoners a chance to leave if they win at the following game:

   Some number of stones are in a pile. The prisoner and the professor alternately remove 1, 2, or 3 stones from the pile. The player who takes the final stone loses.

   Being the gentleman that he is, at the beginning of every game Professor Evil offers his prisoner a choice of going first, going second, or specifying the number of stones in the pile. Once the prisoner announces his choice, the professor then takes out the stones and the game begins.

   (a) Show that if there are 2, 3, or 4 stones on your turn, then you win.
   (b) Show that if there are 5 stones on your opponent's turn, then you win.
   (c) Use induction to show that if there are $4N + 1$ stones in the pile during your opponent's turn, then you win.
   (d) Given the conditions of the game, show that the professor always wins if he plays correctly.